## Busill Jones Primary School Calculation Policy

This policy supports the White Rose Maths Schemes of Learning from Reception to Year 6. Each area of study progresses in line with the National Curriculum (2014) and the EYFS Early Adopter Framework (2020).

This calculation policy should be used to support children in developing a deep understanding of number and calculation, gaining mastery mathematics knowledge.

## Concrete, Pictorial, Abstract (CPA) Approach

Children of all ages are first introduced to new mathematical learning by using real objects (concrete resources). They are offered a 'hands on' experience with manipulatives to support their fundamental knowledge as a foundation for their conceptual understanding. This is then followed by a pictorial representation which reflects the concrete manipulatives previously used. The children then make connections between the concrete resources and the pictorial representations. After sufficient foundation knowledge is gained, the pupils move onto an abstract representation using mathematical notations. To begin with, this concept is used parallel with the pictorial and concrete representations to secure the children's knowledge of all procedures.

These skills are reinforced through all representations being used throughout school, irrespective of the year group.

## Reasoning and Problem Solving

Each lesson, children are exposed to reasoning and problem-solving questions to embed their understanding of the skills gained within the lesson. They use their learning in real-life contexts to solve complex and abstract problems, considering skills gained in previous areas of learning.


|  | Children experiment with combining different Numicon tiles together to find a total or match another piece. | Children use objects and numicon to help them compare objects and to say how many more or less. |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Key <br> Vocabulary | Add More And + Total Make Sum Lots Same Larger Smaller | Take Away Less Left Over <br> Fewer Difference Between <br> equal to/equals - | Same | Share |
| Resources | Numicon Counting Equipment | Counting equipment | Sorting resources | Sorting resources |

## Children are encouraged to develop a mental picture of the calculation to support their understanding.



\section*{Subtraction <br> O-O (take -away) <br> 1 less than a given number up to 20. <br> O-O (comparison e.g. - 'how many <br> more...'; 'how many less...') <br> Compare numbers using language such as 'less than' and 'fewer than' and have a good understanding of 'one less than'. Understanding of numbers to 10 and link this knowledge to subtraction. <br> Begin with numbers to 5 and understand the number bonds using a range of resources and physical objects, encouraging subitising. Move on to larger numbers as children develop a secure understanding. <br> | Multiplication |
| :--- |
| Progression of Calculation | <br> Counting in 2 s and 10 s <br> Beginning to double single-digit numbers. concrete resources. of objects using counting equipment, Numicon, Cuisenaire etc.}

Be able to recall number bonds to 10. Use opportunities to encourage children to recall number bonds e.g. 'there were 5 children on the carpet but 2 have gone to play. There are now 3 children.'

Become exposed to language such as 'double' and 'half' and see this using

Children will experience equal groups


Children begin to record doubles.


## Division

Progression of Calculation
Creating equal groups of a set of objects. Sharing a set of objects
Become exposed to language such as 'double' and 'half' and see this using concrete resources.

Children will understand equal groups and share items out in play and problem solving.


Explore sharing into equal groups and sets with counting equipment, Numicon, Cuisenaire.




Children find a total to an addition by counting on from the first number, leading on to counting on from the larger number.

Begin to use known addition facts such as doubles or bonds to 10 .


Children use numbered lines (to 10 and beyond), bead strings and practical resources to support addition.

$$
1 \bigcap_{6}^{+1} \bigcap_{9}^{+1}+\bigcap_{11}^{+1+1}+1
$$


$8+5=13$
Use Numicon tiles to calculate addition and solve problems e.g. all pairs that make 10. Begin to represent for bonds to 20. Use resources to support understanding of inverses.

## 

complete missing number problem such as
$3+\square=5$ using the part/whole model and resources. Understand what happens when zero is added to a number and embed using number lines and concrete resources.

| $\square$ |  | $\begin{aligned} & 6+4=10 \\ & 4+6=10 \\ & 10-4=6 \\ & 10-6=4 \end{aligned}$ | $\text { (8) } \begin{aligned} & 6+4=10 \\ & (10) \\ & 4+6=10 \\ & 10-4=6 \\ & 10-6=4 \end{aligned}$ | 10 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 6 |  | 4 |
| - | $\bigcirc$ |  |  |  |  |
| - | $\bigcirc$ |  |  |  |  |
| - | $\bullet$ |  |  |  |  |
|  | $\bullet$ |  |  |  |  |
|  | Tens |  |  | Part Whole Model |  |  |


be used to illustrate subtraction including
bridging

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Use related subtraction facts to 20 and use known facts to support other facts e.g. 20-9 by using fact $20-10$

Subtraction to be calculated by crossing out pictures and introduction to bar models and tens frames.


Number lines and other resources should be used to show that subtraction can also mean finding the difference e.g. 9-4 = can also mean 'the difference between 4 and 9 ' and how many jumps they are apart.


Continue to use pictures, diagrams, and a range of resources to calculate and see numbers represented in a range of ways.


Understand what happens when zero is subtracted from a number

Introduction to the part whole model to complete missing number problems e.g. $10-5=$

$10 \times 8=80$
Begin using an array to aid multiplication.


Use Numicon and other objects to aid multiplication and to support understanding
 of doubles.

Children begin to make connections between arrays, number patterns and

counting in $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s.

Children begin to use a number line and hundred square to develop their understanding of multiplication as repeated addition and to record their multiplication of $2 s, 5 s$ and 10 s . They should recognise that if numbers are grouped equally, it is more efficient to count in groups rather than ones.

## How many pencils are there?


https://assets publishing service. gov.uk/gov


Children to use informal recording when ready

## For assessment questions, see Mathematics

## Guidance sections: 1NPV and 1NF

https://assets.publishing.service.gov.uk/governm ent/uploads/system/uploads/attachment data/fil e/897806/Maths guidance KS 1 and 2.pd

|  | Become familiar with the 100 square - quickly find where a number is located. Know that we move to the right and downwards when adding. Use to support adding multiples of 10 . <br> For assessment questions, see Mathematics Guidance sections: 1NPV, 1NF and 1AS https://assets.publishing.service.gov.uk/government/u ploads/system/uploads/attachment data/file/897806/ Maths guidance KS 1 and 2.pdf | The position of the equals symbol should be varied so that children can recognise that calculations can be written in different ways. <br> Become familiar with the 100 square - quickly find where a number is located. Know that we move upwards and left when subtracting. <br> Reasoning should be done verbally. For assessment questions, see Mathematics Guidance sections: 1NPV, 1NF and 1AS https://assets.publishing.service.gov.uk/gover nment/uploads/system/uploads/attachment data/file/897806/Maths guidance_KS_1 and 2.pdf |  |  | ernment/uploads/system/uploads/attachm ent data/file/897806/Maths guidance KS 1 and 2.pdf |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Key <br> Vocabulary | Partition Equals/Same as <br> Addition Number bonds <br> More than Missing number <br> Total/Sum  | Subtraction  <br> Take Away Less Left Over <br> Fewer Difference Between |  |  | Multiplication Groups of Equal <br> Multiply Lots of  <br> Multiplied by Sets of  <br> Multiple Array  | Half Grouping <br> Halve Share <br> Division Equal <br> Dividing  |
| Resources | Numicon Hundred Square <br> Bead Strings Straws <br> Number lines Tens frame <br> Counting equipment Double sided counters | Bead Strings Tens frame <br> Number lines <br> Counting equipment <br> counters Straws <br> Hundred Square Double sided |  |  | Tens frames Straws <br> Arrays Double sided counters <br> Numicon Bead strings <br> Number lines  | Tens frames Straws <br> Arrays Double sided counters <br> Numicon Bead strings <br> Number lines  |
| Children are encouraged to develop a mental picture of the calculation to support their understanding. |  |  |  |  |  |  |
| Year 2 | Addition | Subtraction |  |  | Multiplication | Division |
|  | Progression of Calculation <br> TO+O (not crossing 10) <br> TO+T (not crossing 10) <br> TO+O (bridging through 10) <br> TO + 'teens number' <br> TO+TO (not crossing tens) $0+0+0$ <br> - Understand addition as commutative. <br> - Use the bar model to find missing digits. <br> - Use the inverse to find missing numbers. <br> - Confidence with a strategy to add/subtract, in particular when bridging ten. <br> - Use additive facts to connect numbers e.g. 3 + $4=7$ so $30+40=70$, moving onto recognising $35+40=75$ <br> - Recognise all coins and use this to add to an amount. | Prog <br> TO-O <br> TO-O <br> TO-T <br> TO-T <br> TO-T <br> - Rec com <br> - Rec <br> calcu <br>  <br> 23 <br> Child <br> using <br> Coun | Progression of Calculation <br> TO-O (not crossing 10s) <br> TO-O (crossing 10s) <br> TO-T <br> TO-TO (counting back) <br> TO-TO (counting on) <br> - Recognise that subtraction is not commutative. <br> - Recognise the inverse and use to check calculations using bar models. |  | Progression of Calculation <br> $0 \times 2,5$ and 10 <br> Doubles up to 20+20 <br> - Introduction to the $X$ symbol and use when writing formal calculations. <br> - Recall and reuse facts from times tables, recognising when to use these in problems. <br> - Understand that multiplication is commutative. <br> - Consider introducing counting in 3s towards the end of the year. <br> Express multiplication in a number of ways: | Progression of Calculation <br> Halving 0-40 <br> TO $\div$ ) (using jottings and equipment in context) <br> TO $\div 2,5,10$ (using known facts) <br> TO $\div 2,5,10$ (with remainders) <br> - Recall and use division facts relating to known times tables. <br> - Divide larger numbers by partitioning into tens and ones. <br> - Multiplication and division taught alongside each other to show relationship. |



Counting on e.g. $13+5=$
Children use visual resources to begin to draw their own empty number
lines to support
00000000000000000000
00008 008 000000008 ce10.00d calculation.

## Partitioning

Children use known addition facts such as doubles or bonds to 20 to add more quickly or derive new facts up to 100. E.g. Use knowledge that $4+3=7$ to know $40+30=70$

Children use knowledge of place value to partition two digit numbers. Progression begins with adding tens then ones, progressing to 2 digit add tens add ones.

$$
\begin{aligned}
& 40+20+5+3=60+8=68 \\
& 40+5+20+3=60+8=68 \\
& 45+23=60+8=68
\end{aligned}
$$

## Children to

think
flexible

about numbers and partition in different ways e.g. $23=20+3$ and $23=10+13$

Use partitioning knowledge to support calculation using number lines and hundred squares.
Children to become more efficient by adding the units in one jump (by using the known fact $4+3$ $=7$ ).
Children to be supported in understanding that bridging through ten can lead to more efficient calculation. E.g.
37+15 = 52


Children use visual resources to begin to draw their own empty number lines to support calculations.
Children first counting back in tens and ones.
$47-23=24$


Children then become more efficient by subtracting the units in one jump (by
using
$47-23=24$
the
known
fact 7 -

$3=4$ ).
Children move to subtracting the tens in one jump and the units in one jump. Use a variety of methods when subtracting through ten.

Use a range of resources and representations to make links and recognise numbers shown in a variety of ways.

## Repeated addition

5 times 3 is $5+5+5=15$ or 3
lots of 5 or $5 \times 3$
Although children must understand that multiplication is commutative, it must be taught that:
The 'start' number (the operand) will be the group size; the 'change' number (multiplicand) will be the number of times the group is repeatedly added.

| $\left.\begin{array}{l} 8 \\ 8 \\ 8 \\ 8 \\ 8 \end{array}\right)$ | $\frac{0000}{0000}$ |
| :---: | :---: |
| $12=3 \times 4$ | $12=4 \times 3$ |

Repeated addition can be shown easily on a number line and empty number line:
$5 \times 3=5+5+5$

and on a bead bar:
$5 \times 3=5+5+5$


Links should also be made to money.


And to 5 minute intervals on a clock face.
e.g. 3 lots of 5 minutes $=15$ minutes. Arrays
Children should be able to model a multiplication calculation using an array. This knowledge will support with the understanding of commutativity and the development of the grid method. Links can be made from all of these resources to division as the inverse of multiplication.

Sharing equally without remainders
6 sweets shared between 2 people, how many do they each get?


## Grouping

There are 6 sweets, how many people can have 2 sweets each?


Repeated subtraction using a number line or bead bar
$12 \div 3=4$

仿


Using symbols to stand for unknown numbers to complete equations using inverse operations
$£ \div 2=4 \quad 20 \div r=4 \quad £ \div r=4$

Relate division to fractions e.g. $40 \div 2=20$ 20 is half of 40 . Use half and double alongside one another.

## For assessment questions, see Mathematics

 Guidance sections: 2MDhttps://assets.publishing.service.gov.uk/gov ernment/uploads/system/uploads/attachm

|  | Children continue to use visual resources to support bridging through ten such as bead strings and Numicon. <br> Children to use knowledge of number facts to efficiently add 3 one digit numbers e.g. $7+6+3$ <br> $=$. Children should look for number bonds to 10 or double to solve easily. This will improve knowledge of commutativity. <br> Continue to deepen knowledge of links to subtraction and to solve missing number problems. <br> For assessment questions, see Mathematics Guidance sections: 2NPV, 2NF and 2AS <br> https://assets.publishing.service.gov.uk/government/u ploads/system/uploads/attachment data/file/897806/ Maths guidance KS 1 and 2.pdf |  <br> 65 IIIIIIIDXXX XPSEX $\square$ $65-28=37$ <br> Children should be able to solve problems with missing addends and <br> recognise when to use the inverse to solve. <br> Continue to deepen knowledge of links to addition and to solve missing number problems. <br> For assessment questions, see Mathematics Guidance sections: 2NPV, 2NF and 2AS <br> https://assets.publishing.service.gov.uk/ government/uploads/system/uploads/at tachment data/file/897806/Maths guid ance KS 1 and 2.pdf | Use a range of equipment such as Numicon to show alternative calculations. <br> Recall or derive multiplication facts for the $2,5,10$ times tables. <br> For assessment questions, see Mathematics Guidance sections: 2NPV, 2NF and 2MD <br> https://assets.publishing.service.gov.uk Lgovernment/uploads/system/uploads Lattachment data/file/897806/Maths guidance KS 1 and 2.pdf | ent data/file/897806/Maths guidance KS 1 and 2.pdf |
| :---: | :---: | :---: | :---: | :---: |
| Key <br> Vocabulary |   <br> Partition Equals/Same as <br> Addition Number bonds <br> More than Missing number <br> Total/Sum Inverse <br> Commutative Calculate <br> Strategy Addend | Subtraction Less <br> Take Away Left Over <br> Fewer Difference Between <br> Equal to/Equals Addend <br> How many more  | Row <br> Column <br> Multiplication Table <br> Times <br> Repeated addition | Divided by Divided into Equal groups of |
| Resources | Numicon Base 10 <br> Bead Strings Arrow cards <br> Number lines Hundred Square <br> Counting equipment  | Bead Strings Hundred Square Numberlines Counting equipment Cuisenaire | Cuisenaire $\quad$ Clocks  <br> Numicon  <br> Arrays  <br> Bead Strings  | Bead Strings Arrays |



Children find a total to an addition by counting on from the first number, leading on to counting on from the larger number.

Begin to use known addition facts such as doubles or bonds to 10 .


Children use numbered lines (to 10 and beyond), bead strings and practical resources to support addition.

$$
\overbrace{6} \overbrace{8}^{+1++10+1}+\bigcap_{0}^{+1+1}+1
$$

$$
8+5=13
$$

Use Numicon tiles to calculate addition and solve problems e.g. all pairs that make 10. Begin to represent

Subtraction
Progression of Ca
0-O (take-away)
O-O (comparison and difference)
Any subtraction from 20
-Derive and use additive facts and make connections between these and subtraction problems.

- Subtract multiples of 10 using the
vocabulary 'one ten, two tens, three tens'
longside 10, 20, 30.
- Addition and subtraction to be taught
alongside each other so that children can see the relationship between the two.
- Subtract using Base

Ten/straws and understand regrouping one ten for ten ones.

Begin with physical objects being taken away.

6-3=3
Subtract by counting back, using a number line/bead string. Bead strings or bead bars can be used to illustrate subtraction including bridging


## $000000-\infty 0 \sim$

Use related subtraction facts to 20 and use known facts to support other facts e.g. 20-9 by using fact $20-10$

Subtraction to be calculated by crossing out pictures and introduction to bar models and tens frames.


Number lines and other resources should be used to show that subtraction can also mean finding the difference e.g. 9-4 = can also mean 'the difference between 4 and 9 ' and how many jumps they are apart.

Learn to recognise $2 \mathrm{p}, 5 \mathrm{p}$ and 10 p coins and apply their knowledge of multiples when using these. E.g. calculating how many coins to give to pay for an item. Count in tens to add and subtract (see addition and subtraction).

They will count in 2 s and 10 s and begin to count in 5 s mentally, using a number line to count jumps, using Numicon, using sets of counters.

They will work on practical problem solving activities involving equal sets or groups.

$10 \times 8=80$
Begin using an array to aid multiplication.


Use Numicon and other objects to aid multiplication and to support understanding of doubles.

Children begin to make connections between arrays, number patterns and


Progression of Calulation
Halving 0-20

## $0-20 \div 0$

- Working practically to share and group.
- Children are not expected to divide and calculate formally.


## Sharing

(0000 00 ?
There are 20 apples altogether.
They are shared equalll between 5 bags.
How many apples are in each bag?

## Grouping

Encourage children to use multiple knowledge and count in multiples.
Link to repeated subtraction using a number line. Use fixed groups/resources (e.g. Numicon) to show relationship between multiplication and division

0.000
-00000-00000-00000-00000-
 There are 20 apples altogether. They are put in bags of 5 .
How many bags are there



Children to use informal recording when ready
For assessment questions, see Mathematics Guidance sections: 1NPV and 1NF
https://assets.publishing.service.gov.uk/governm ent/uploads/system/uploads/attachment data/fil e/897806/Maths guidance KS 1 and 2.pdf

|  | for bonds to 20. Use resources to support understanding of inverses. <br> Complete missing number problem such as $3+\square=5$ using the part/whole model and resources. Understand what happens when zero is added to a number lines and concrete resources. <br> Use tens frames to embed understanding of bridging 10. E.g. $9+3=12$. Take one from the 3 and give it to the 9 to turn 9 into 10 so it is now $10+2=12$. <br> Become familiar with the 100 square - quickly find where a number is located. Know that we move to the right and downwards when adding. Use to support adding multiples of 10 . <br> For assessment questions, see Mathematics Guidance sections: 1NPV, 1NF and 1AS <br> https://assets.publishing.service.gov.uk/government/u ploads/system/uploads/attachment data/file/897806/ Maths guidance KS 1 and 2.pdf | Continue to use pictures, diagrams, and a range of resources to calculate and see numbers represented in a range of ways. <br> Understand what happens when zero is subtracted from a number. <br> Introduction to the part whole model to complete missing number problems e.g. $10-5=$ <br> The position of the equals symbol should be varied so that children can recognise that calculations can be written in different ways. <br> Become familiar with the 100 square - quickly find where a number is located. Know that we move upwards and left when subtracting. <br> Reasoning should be done verbally. For assessment questions, see Mathematics Guidance sections: 1NPV, 1NF and 1AS https://assets.publishing.service.gov.uk/gover nment/uploads/system/uploads/attachment data/file/897806/Maths guidance KS 1 and 2.pdf | Children begin to use a number line and hundred square to develop their understanding of multiplication as repeated addition and to record their multiplication of $2 \mathrm{~s}, 5 \mathrm{~s}$ and 10 s . They should recognise that if numbers are grouped equally, it is more efficient to count in groups rather than ones. <br> For assessment questions, see Mathematics Guidance sections: 1NPV and 1NF https://assets.publishing.service.gov.uk/gov ernment/uploads/system/uploads/attachm ent data/file/897806/Maths guidance_KS 1 and 2.pdf |  |
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| Resources | Numicon Hundred Square <br> Bead Strings Straws <br> Number lines Tens frame <br> Counting equipment Double sided counters | Bead Strings Tens frame <br> Number lines <br> Counting equipment <br> counters Straws <br>  Double sided | Tens frames Straws <br> Arrays Double sided counters <br> Numicon Bead strings <br> Number lines  | Tens frames Straws <br> Arrays Double sided counters <br> Numicon Bead strings <br> Number lines  |


| Year 3 | Addition |
| :---: | :---: |
|  | Progression in Calculation |
|  | TO+TO (bridging through 10s, not crossing 1 TO+TO |
|  |  |
|  | HTO+TO |
|  | HTO+HTO |
|  | - Know and use complements to 100 , especia when calculating money problems and findi change. |
|  | - Recognise the inverse and use to check calculations |
|  | Know place value to 3 digits. |
|  | $\begin{array}{l\|l\|l} 8 & 5 & 4 \\ \hline \end{array}=\begin{aligned} & 800 \\ & \hline \end{aligned}$ |

Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.

Children will count on from the largest number irrespective of the order of the calculation and partition the $2^{\text {nd }}$ number only. E.g.

$$
38+86=124
$$



## Compensation

Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate.


## Expanded methods of addition.

Adding the least significant digits first.
Diennes/base 10 materials must be used to support understanding of columnar methods of addition. - Up to 3 digit columnar addition.

## Subtraction $\quad$ Multiplication

| Progression of Calculation | Progression of Calculation |
| :--- | :--- |

Empty Number Line
HTO-TO (crossing100s)
HTO-HTO
Expanded Column Method
HTO-HTO
Stage 1 : no exchange
Stage 2: Exchange $T$ to $O$
Stage 3: Exchange H to T

- Recognise the inverse and use to check calculations

Children will continue to use empty number lines with increasingly large numbers.


Partitioning and decomposition

- Partitioning -

demonstrated using
arrow cards
- Decomposition Diennes/base 10 materials must be used to support understanding of columnar methods of subtraction. Up to 3 digit columnar subtraction.

NOTE - When solving the calculation 89 57, children should know that 57 DOES NOT EXIST AS AN AMOUNT it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89 .

Ox (2,3,4,5,8,10)
(1-20) x $(2,3,4,5,8,10)-$ beyond 20
when confident.
Multiply 1 or 2 digit numbers by 10 and 100.

- Understand the commutative property of multiplication

Children will continue to use:

## Repeated Addition

4 times 6 is $6+6+6+6=24$ or 4 lots of 6 or $6 \times 4$

The 'start' number (the operand) will be the group size; the 'change' number (multiplicand) will be the number of times the group is repeatedly added.

Children should continue to use number lines or bead bars to support their understanding of repeated addition
$\sim_{0}^{6}$
-66600 -06600-000600-006600 ${ }^{6}$

\section*{| 00 | 0 | 0 | 0 | 000 |
| :--- | :--- | :--- | :--- | :--- |}

## Scaling

Children must be given opportunities to answer problem-solving questions related to the 'Scaling' model of multiplication.
E.g. Find a ribbon that is 4 times as ng as the blue ribbon.


## Division

## Progression of Calculation

TO $\div(2,3,4,5,8,10)$ - answers between 0 and 10.
TO $\div(2,3,4,5,8,10)$ - answers between 1 and 10 with remainders.-

- Divide 100 into $2,4,5$ or 10 equal parts by practising counting multiples of $10,20,25$ or 50.

Use of both methods:

- Grouping
- Sharing

Children will continue to use:
Repeated Subtraction

## \section*{Counters} <br> $\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \varnothing \varnothing \varnothing \varnothing$

Using a number line
Children will use an empty number line to support their colculation.
24-4:6
20
Children should be allowed to move
forwards or backwards on the number line depending on familiarity with times tables.

## Remainders

Children should move onto calculations involving remainders, jumping forwards or backwards on the number line. E.g. $13 \div 4=$ 13;4=3r1


$\qquad$

Arrays
Arrays and known facts can be used to support children in answering missing number calculations.
E.g. $24 \div £=6$
$20+5$
$40+8$
$60+13=73$

## Estimation

Children must begin to make sensible estimates of their answers.

## When children are confident with adding two

 addends, they should move on to more than two, using their known facts to support adding in an efficient order e.g. doubles and number bonds.

Children to move onto formal column methods if place value is secure.
TO + TO

HTO + TO

HTO + HTO


Bar Model


Ensure a range of methods are used to aid children when problem solving

Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.

## $\square \square_{\square} \square_{\square} \square_{\square}$  $9 \times 4=36$

Arrays can also be used to support the associative law of multiplication. E.g.

| - | 0000 | - |
| :---: | :---: | :---: |
|  | 000 | - |
|  |  |  |
|  | - |  |
|  |  |  |
|  |  |  |
|  |  |  |

$4 \times 7+4 \times 7=8 \times 7$

## Missing Number Problems

Children to use symbols to stand for unknown numbers to complete equations using inverse operations
$£ \times 5=20$ estimates of their answers

## Bar Model

| 315 |  | $315-185=?$ |
| :---: | ---: | ---: |
| 185 | $?$ | $185+?=315$ |

Ensure a range of methods are used to aid children when problem solving.

$£ x$ š $=32$
18
Partitioning
$38 \times 5=(30 \times 5)+(8 \times 5)$
$=150+40$
$=190$
Children must begin to make sensible estimates of their answer.

Children will know 3, 4, 8 multiplication facts. They will use doubling and resources to see the links between the 2,4 , and 8 multiplication tables.


## Bar Model

E.g. 4 children go to the cinema. They each pay $£ 15$. Ho much do they spend


Children should be encouraged to make sensible estimates of their answers.

## Missing Number Problems

Children will use symbols to stand for unknown numbers to complete equations using inverse operations
$26 \div 2=£$
$24 \div r=12$

Place Value Grid altogether?

|  | $\square$ <br> 164 <br> $265+164=429$ <br> For assessment questions, see Mathematics Guidance sections: 3NPV, 3NF and 3AS https://assets.publishing.service.gov.uk/govern ment/uploads/system/uploads/attachment data /file/897806/Maths guidance KS 1 and 2.pdf | For assessment questions, see <br> Mathematics Guidance sections: 3NPV, <br> 3NF and 3AS <br> https://assets.publishing.service.gov.uk/ government/uploads/system/uploads/at tachment data/file/897806/Maths guid ance KS 1 and 2.pdf | For assessment questions, see <br> Mathematics Guidance sections: <br> 3NPV, 3NF and 3MD <br> https://assets.publishing.service.gov.uk <br> /government/uploads/system/uploads <br> /attachment data/file/897806/Maths <br> guidance KS 1 and 2.pdf | For assessment questions, see Mathematics Guidance sections: 3NPV, 3NF and 3MD https://assets.publishing.service.gov.uk/gov ernment/uploads/system/uploads/attachm ent data/file/897806/Maths guidance KS 1 and 2.pdf |
| :---: | :---: | :---: | :---: | :---: |
| Key <br> Vocabulary | All previous words <br> Increase - get larger <br> Addend - a number to be added to another <br> Augmentation - increase a quantity or measure <br> by another quantity. <br> Commutative - numbers can be added in any order. <br> Complement - in addition, a number and its complement make a total <br> Exchange - change a number for another of an equal value. <br> Partitioning - split a number into parts. <br> Subitise - instantly recognise the number of objects in a small group. <br> Sum - the result of an addition <br> Total - the aggregate or the sum found by addition. <br> Ones/Tenths/Hundreds Boundary - The boundary to the next PV column - knowing when to exchange. <br> Inverse - the opposite in effect. | All previous words <br> Decrease - get smaller <br> Difference - the numerical difference between two numbers found by comparing each quantity <br> Exchange - change a number or expression for another of equal value <br> Minuend - a quantity or number from which another is subtracted <br> Partitioning - splitting a number into parts <br> Reduction - subtraction as take away Subtrahend - a number subtracted from another <br> Inverse - the opposite in effect. | All previous words <br> Factorise - finding what to multiply to get an expression. <br> Array - an ordered collection of resources in rows and columns. <br> Commutative - numbers that can be multiplied in any order. <br> Exchange - change a number for another of an equal value. <br> Factor - a number that multiplies with another to make a product. <br> Multiplicand - a number to be multiplied by another. <br> Partitioning - splitting a number into parts. <br> Product - the result of multiplying one number by another. <br> Scaling - enlarging or reducing a number by a given amount. <br> Inverse - the opposite in effect. | All previous words <br> Dividend - the number that is divided. <br> Divisor - the number by which another is divided <br> Factor - a number that multiplies to make another product. <br> Quotient - the result of a division <br> Remainder - the amount left over when the divisor is not a factor of the dividend. <br> Scaling - enlarging or reducing a number by a given amount. <br> Inverse - the opposite in effect. |
| Resources | Diennes/Base 10/Big Base <br> Calculation Mat <br> Arrow Cards <br> Part-Whole Model <br> Bar Model <br> Number Shapes (Numicon) <br> Cubes <br> Tens Frames | Diennes/Base 10/Big Base <br> Calculation Mat <br> Arrow Cards <br> Part-Whole Model <br> Bar Model <br> Number Shapes (Numicon) <br> Cubes <br> Tens Frames | Cuisenaire <br> Diennes/Base 10/Big Base <br> Numicon <br> Counters <br> Multiplication Squares <br> Bead Strings <br> Empty Number Line <br> Number Tracks/Number Lines | Cuisenaire <br> Diennes/Base 10/Big Base <br> Numicon <br> Counters <br> Multiplication Squares <br> Bead Strings <br> Empty Number Line |


|  | Bead Strings |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | Number Tracks/Number Lines/Empty number |  |  |  |
| line |  |  |  |  |
| Place value counters/Place value grid/Place |  |  |  |  |
| Value Sliders |  |  |  |  |
| Double sided counters |  |  |  |  |
| Bar Models Tens frame (scaling known facts) | Bead Strings <br> Number Tracks/Number Lines/Empty <br> number line <br> Place value counters/Place value <br> grid/Place Value Sliders <br> Double sided counters <br> Bar Models Tens frame (scaling known <br> facts) | Place Value Counters/Place value <br> grid/Place Value <br> Calculation Mat | Plamber Tracks/Number Lines <br> Place Value Sliders | Bar Models |




Finally using Place Value Counters which make it necessary for children to understand 'a one to many' representation of value.


Revert back to expanded method for children who develop misconceptions.

Ensure a range of methods are used to aid children when problem solving.


## Bar Model

On one day Peter ran 6,860 metres. The next day, Peter ran 5,470 metres. How far does he run altogether?

$$
\begin{array}{r}
6141 \\
7 / 64 \\
-\quad 86 \\
\hline 668
\end{array}
$$

As in addition, a progression of resources should be used to support the development of concept and understanding alongside a written method.


When completing three digit money calculations children will know that decimal points should line up under each other.

$$
£_{8,95}=8+0.9+0.05
$$

£8.95 $=8+0.9+0.05$
$\underline{-£ 4.38}-\underline{-4+0.3+0.08}$

$$
\begin{aligned}
= & 8+0.8+0.15 \\
& -\frac{4+0.3}{4+0.08}+0.08
\end{aligned}
$$

$$
=£ 4.57
$$

Ensure a range of methods are used to aid children when problem solving.


## Bar Model

There are 3,160 counters in a bag. 1,226 are red counters and the red are blue. How many are blue counters?

| $\times$ | 20 | 3 |
| :---: | :---: | :---: |
|  | 160 | 24 |

Children will continue to use arrays where appropriate leading into the grid method of multiplication.


Use of resources, arrays and the grid method to support the links between division and multiplication.

Use of arrays to support understanding of the laws of multiplication.

Expanded column method with


## representations

## Children must understand the

## distributive law of multiplication and

 represent this in mixed operation calculations (as above).Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of $10 \mathrm{~s}, 5 \mathrm{~s}, 2 \mathrm{~s}$ and 1 s - numbers with which the children are more familiar. $72 \div 5$
 027121722 27323742475257626772


Then onto the vertical method: Short Division TO $\div$ U

## E.g. $72 \div 3$ =



## Leading to Short Division HTO $\div \mathbf{O}$ (as

 above)Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2. Children need to consider how to express their quotient and round up or down accordingly. They should make sensible decisions about rounding up or down after division.


|  | Number Tracks/Number Lines/Empty number | Number Tracks/Number Lines/Empty | Place Value Counters/Place value |
| :--- | :--- | :--- | :--- | :--- |
| line |  |  |  |
| grid/Place Value |  |  |  |
| Place value counters/Place value grid/Place |  |  |  |
| Value Sliders |  |  |  |
| Double sided counters |  |  |  |
| Bar Models Tens frame (scaling known facts) | Place value counters/Place value <br> grid/Place Value Sliders <br> Couble sided counters <br> Bar Models Tens frame (scaling known <br> facts) | Place Value Sliders <br> Bar Models | Calue |

Children are encouraged to develop a mental picture of the calculation to support their understanding.

| Year 5 | Addition | Subtraction | Multiplication | Division |
| :---: | :---: | :---: | :---: | :---: |
|  | Progression of Calculation <br> - Addition of numbers beyond 1000s <br> - Addition of numbers with up to three decimal places. <br> - Addition of numbers with up to 4 digits and decimals with various missing place-holders. <br> - Using a bar model to find missing amounts and to support problem solving. <br> - Scale known additive facts, both within 10 and bridging 10 when adding/subtracting decimals. <br> E.g. $0.5+0.4=0.9$ or $0.8+0.6=1.4$ <br> Also use this knowledge with complements to 100 - scale to complements to 1. | Progression of Calculation <br> - Subtract at least 4 digit numbers and up to 3 decimal places. <br> - Larger numbers involving decimals including money and measures. <br> - Use bar models to support subtraction. <br> - Scale known additive facts, both within 10 and bridging 10 when adding/subtracting decimals. E.g. $0.9-0.5=0.4$ and $1.5-0.6=0.9$ <br> Also use this knowledge with complements to 100 - scale to complements to 1 . $\text { E.g. } 100-65=$ <br> 35 so $1-0.65=$ <br> 0.35 <br> 100 grids may also be used to show complements to 1 . <br> Empty Number Lines and Decimals | Progression of Calculation <br> - ThHTO x 0 <br> - HTO x O <br> - TO x TO <br> - Use a bar model to support problem solving in multiplication. <br> - Multiply by 10, 100 and 1000 and understand how this affects a number's place value. <br> Area Model <br> Area will continue to support understanding of short and long multiplication, using concrete and pictorial resources to support understanding. | Progression of Calculation <br> - TO $\div 0$ <br> - HTO $\div \mathrm{O}=$ TO <br> - НTO $\div \mathrm{O}=\mathrm{HTO}$ <br> - THTO $\div \mathrm{O}=$ <br> - Using a bar model to support problem solving in division <br> -Divide by 10, 100 and 1000 and understand how this affects a number's place value. <br> Grouping to support Short Division Children will use concrete and pictorial resources to support grouping when using short division with progressively larger numbers. |



|  | Augmentation - increase a quantity or measure by another quantity. <br> Commutative - numbers can be added in any order. <br> Complement - in addition, a number and its <br> complement make a total <br> Exchange - change a number for another of an equal value. <br> Partitioning - split a number into parts. <br> Subitise - instantly recognise the number of objects in a small group. <br> Sum - the result of an addition <br> Total - the aggregate or the sum found by addition. <br> Ones/Tenths Boundary - The boundary to the next PV column - knowing when to exchange. | Difference - the numerical difference between two numbers found by comparing each quantity <br> Exchange - change a number or expression for another of equal value <br> Minuend - a quantity or number from which another is subtracted <br> Partitioning - splitting a number into parts <br> Reduction - subtraction as take away <br> Subtrahend - a number subtracted from another | Prime Factor - the factors of a number that are prime numbers. <br> Array - an ordered collection of resources in rows and columns. <br> Commutative - numbers can be multiplied in any order. <br> Exchange - change a number for another of an equal value. <br> Factor - a number that multiplies with another to make a product. <br> Multiplicand - a number to be multiplied by another. <br> Partitioning - splitting a number into parts. <br> Product - the result of multiplying one number by another. <br> Scaling - enlarging or reducing a number by a given amount. | Factor - a number that multiplies to make another product. <br> Quotient - the result of a division Remainder - the amount left over when the divisor is not a factor of the dividend. <br> Scaling - enlarging or reducing a number by a given amount. |
| :---: | :---: | :---: | :---: | :---: |
| Resources | Double sided counters <br> Place value counters <br> Base Ten <br> Calculation Mat <br> Empty Number Line <br> Arrow Cards <br> Place value grid <br> Place Value Sliders <br> Bar Models Tens frame (scaling known facts) | Double sided counters <br> Place value counters <br> Base Ten <br> Calculation Mat <br> Empty Number Line <br> Arrow Cards <br> Place value grid <br> Place Value Sliders <br> Bar Models | Place Value Counters <br> Calculation Mat <br> Empty Number Line <br> Arrays <br> Place Value Sliders <br> Bar Models | Place Value Counters Calculation Mat Empty Number Line Place Value Sliders Bar Models |

## Children are encouraged to develop a mental picture of the calculation to support their understanding.

| Year 6 | Addition | Subtraction | Multiplication | Division |
| :---: | :---: | :---: | :---: | :---: |
|  | Progression in Calculation <br> - Addition of numbers with any number of digits. <br> - Addition of two or more numbers with up to 3 decimal places (including problems involving money). <br> - Addition of two or more numbers with at least 4 digits of various sizes and varied decimal places. (E.g. $401.2+26.85+13=$ ) <br> - Using the bar model to support addition and problem solve. <br> - Use a given additive calculation to derive or complete a related calculation, using arithmetic and place value knowledge and inverse operations. <br> Children will continue to develop procedural fluency of the column method. Children to use | Progression in Calculation <br> Refine Year 5 <br> Subtracting with increasingly more complex numbers with up to 3 decimal places (including problems involving money). <br> Using the bar model to support subtraction. <br> Difference between two negative integers. <br> Difference between positive and negative integers. <br> Solve problems relating to subtraction of any single place value part from a number: | Progression of Calculation <br> THHTOxO <br> HTOXTO <br> $0 \times 0 . t h$ <br> Multiply by up to 2 decimal places. <br> Understand the relationship between <br> powers of 10 and apply this. <br> Use bar models to support <br> multiplication. <br> Use a given multiplicative calculation to derive or complete a related calculation, using arithmetic and place value knowledge and inverse operations. <br> Children may continue to use the area model as necessary to maintain understanding of place value when multiplying larger numbers and | Progression of Calculation <br> THTO $\div \mathrm{O}$ (with decimal and fraction remainders, depending on context) $\text { НTO } \div \text { TO }$ <br> THTO $\div$ TO <br> 0.t $\div 0$ <br> TO.t $\div 0$ <br> All methods below will be viewed as formal methods. <br> Children should understand how to express remainders as whole number remainders, rounded numbers, fractions and decimals and select the most appropriate way to express the remainder in the context of the question. |



Children will use a range of representations to support additions, including when solving problems, for example:


## For assessment questions, see Mathematics

 Guidance sections: 6NPV and 6AS/MDhttps://assets.publishing.service.gov.uk/govern ment/uploads/system/uploads/attachment data /file/897806/Maths guidance KS 1 and 2.pdf
$381,920-900=$

381,920 - $\qquad$ $=380,920$

## Empty Number Line for Negative

## Numbers

An empty number line will be used to show differences between negative numbers and positive and negative numbers.
E.g. What is the difference between 40 and -35?


Children will use a range of representations to support subtraction, including when solving problems. For


## example:

For assessment questions, see
Mathematics Guidance sections: 6NPV and 6AS/MD
https://assets.publishing.service.gov.uk/ government/uploads/system/uploads/at tachment data/file/897806/Maths guid ance KS 1 and 2.pdf
understanding the process of finding partial products.
Move understanding to column method as soon as secure

## FORMAL METHOD - Long

## Multiplication

Children to move to a
more traditional
compact method of long multiplication if they are confident in their understanding of the expanded method

Children will use a range of representations to support subtraction, including when solving problems. For example:


| $\times$ | 200 | 30 | 4 |
| :---: | :---: | :---: | :---: |
| 30 | 6,000 | 900 | 120 |
| 2 | 400 | 60 | 8 |

For assessment questions, see Mathematics Guidance sections: 6NPV and 6AS/MD
https://assets.publishing.service.gov.uk government/uploads/system/uploads /attachment data/file/897806/Maths guidance KS 1 and 2.pdf

| 22 <br> 792 <br> -720 <br> 72 |  |
| ---: | ---: |
| -72 | $20 \times 36$ |
| 0 |  |

Short Formal Method


| Chunking with | 12.5 |  |
| :---: | :---: | :---: |
| Decimals | $7 \longdiv { 8 7 . 5 }$ |  |
|  | 70.0 | $10 \times 7$ |
|  | 17.5 |  |
|  | 14.0 | $2 \times 7$ |
|  | 3.5 |  |
|  | 3.5 | $0.5 \times 7$ |

Answer: 12.5

## Using a bar model to support division:

Paul and David hire a car together at a cost of $£ 297.50$. Paul pays 6 times more than David. How much does David pay?

| $£ 297.50$ |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  | $?$ |

## For assessment $q$ uestions, see

Mathematics Guidance sections: 6NPV and 6AS/MD
https://assets.publishing.service.gov.uk/gov

|  |  |  |  | ernment/uploads/system/uploads/attachm ent data/file/897806/Maths guidance KS 1 and 2.pdf |
| :---: | :---: | :---: | :---: | :---: |
| Key <br> Vocabulary | Increase - get larger <br> Addend - a number to be added to another <br> Augmentation - increase a quantity or measure by another quantity. <br> Commutative - numbers can be added in any order. <br> Complement - in addition, a number and its <br> complement make a total <br> Exchange - change a number for another of an equal value. <br> Partitioning - split a number into parts. <br> Subitise - instantly recognise the number of objects in a small group. <br> Sum - the result of an addition <br> Total - the aggregate or the sum found by addition. <br> Ones/Tenths Boundary - The boundary to the next PV column - knowing when to exchange. | Decrease - get smaller <br> Difference - the numerical difference between two numbers found by comparing each quantity <br> Exchange - change a number or expression for another of equal value <br> Minuend - a quantity or number from which another is subtracted <br> Partitioning - splitting a number into parts Reduction - subtraction as take away Subtrahend - a number subtracted from another | Factorise - finding what to multiply to get an expression. <br> Prime Factor - the factors of a number that are prime numbers. <br> Array - an ordered collection of resources in rows and columns. <br> Commutative - numbers can be multiplied in any order. <br> Exchange - change a number for another of an equal value. <br> Factor - a number that multiplies with another to make a product. <br> Multiplicand - a number to be multiplied by another. <br> Partitioning - splitting a number into parts. Product - the result of multiplying one number by another. <br> Scaling - enlarging or reducing a number by a given amount. | Dividend - the number that is divided. <br> Divisor - the number by which another is divided <br> Factor - a number that multiplies to make another product. <br> Quotient - the result of a division <br> Remainder - the amount left over when the divisor is not a factor of the dividend. <br> Scaling - enlarging or reducing a number by a given amount. |
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